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DETERMINATION OF FLOW RATES

TRANSONIC TURBINE TEST RIG

by

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UNITED STATES NAVAL POSTGRADUATE SCHOOL
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TABLE OF CONTENTS

TABLES: Page

FIGURES: Page

1. SUMMARY:

This Technical Note establishes the relations for determining the flow rates of the Transonic Turbine Test Rig at the Propulsion Laboratory of the Department of Aeronautics, U. S. Naval Postgraduate School.

Experimental results of flow nozzle calibration tests and of partial range plenum labyrinth seal leak tests are presented. In addition, formulas for the estimation of shaft seal leakage are included. The equations that must be used to determine the flow rate through the turbine are listed in Table IV.

The contents of this Technical Note will be incorporated in a forthcoming thesis by the author.

2. SYMBOLS:

A - Area (in^2)

b - heighth of labyrinth seal intertooth chamber (in)

C - constant in sharp-edged orifice equation

D_1 - diameter of pipe upstream of orifice (nozzle) (in.)

D_2 - diameter of orifice-minimum diameter of nozzle (in.)

g - acceleration due to gravity (32.17 lbm ft/lbs sec^2)

h_w - differential pressure (in H_2O)

K - discharge coefficient (dimensionless)

n - number of throttlings in labyrinth seal (dimensionless)

P_1 - pressure upstream of orifice or nozzle (in. Hg)

p_o - total pressure at labyrinth seal inlet (psia)

p_1 - pressure upstream of orifice or nozzle (psia)

p - static pressure at labyrinth seal discharge (psia)

R - gas constant for air (53.35 ft-lb/lbm^{o}R)

R_e - Reynold's Number based on D_2 (dimensionless)

r - overall pressure ratio of labyrinth seal (p/p_o)

s - labyrinth seal intertooth chamber width (in)

t - labyrinth seal tooth thickness (in)

t - temperature (^{o}F)

T_1 - temperature upstream of orifice or nozzle (^{o}R)

T_o - total temperature at labyrinth seal inlet (^{o}R)

W - mass flow rate (lbm/hr)

w - mass flow rate (lbm/sec)

$w^* = \frac{w\sqrt{To}}{A\ po}\sqrt{R/g}$: non-dimensional flow rate

$X = Re \times 10^{-6}$

Y_1 - expansion factor, orifice or nozzle

α - coefficient of thermal expansion, orifice or nozzle flow ($^{o}F^{-1}$)

α - coefficient of discharge, labyrinth seal leakage (dimensionless)

$\beta \equiv D_2/D_1$

γ - ratio of specific heats = 1.4

γ^* - carryover factor, labyrinth seal leakage

δ - labyrinth seal tooth clearance

Δ - tolerance designator, signifies percent

ω - labyrinth seal pressure ratio function (dimensionless)

ρ_1 - mass density (lbm/ft^3)

ζ - see equation (15)

Subscripts

ℓ - linear

n - nozzle

o - orifice

x - refers to X

y - refers to Y_1

3. INSTALLATION:

The installation of the Transonic Turbine Test Rig is shown schematically in Figure 1. Features of the rig pertinent to this report are:

(a) the flow measurement nozzle (Figure 2);

(b) the removable standard orifice installation (see note 1, Figure 1 and Figure 3);

(c) the plenum labyrinth leak rate measurement standard orifice (2 in. pipe orifice, Figure 1 and Figure 4);

(d) the turbine plenum labyrinth seals (Figure 5);

(e) the shaft labyrinth seal (Figure 5); and

(f) the exhauster installation, including associated piping (Figure 1).

This turbine can be operated exhausting either to atmospheric pressure or to a partial vacuum in the hood, thence through the exhauster. Figure 1 shows all possible piping arrangements, including the removable calibration standard orifice installation and the exhauster with its associated piping.

The flow rate through the turbine blading during operation is the flow measured by the flow nozzle less the plenum labyrinth seal leakage. Since there is insufficient piping upstream of the nozzle to insure undisturbed flow, the nozzle was calibrated using a standard orifice installation, so that accurate flow rates during turbine operation can be obtained from the nozzle flow. Plenum labyrinth seal leak rates can not be measured during turbine operation. Hence, such leakage is determined from separate tests in which all flow to the plenum passes through the two inch line standard orifice and the only exit for the flow is through the labyrinth seals, since the turbine discharge is blocked by means of a special cover plate.

In analyzing exhauster performance, the flow rates to the exhauster from the turbine hood and through the exhauster nozzle are required. The former consists of the flow rates through the turbine, through the labyrinth

seals and through the shaft seals. There are no provisions for actual measurement of flow rates through the shaft seal or through the exhauster nozzle. However, with the entire flow of the Allis Chalmers compressor routed to the turbine rig, the flow through the exhauster nozzle is the flow rate delivered by the compressor, determined as specified in Reference 1, minus the flow rate measured by the flow nozzle. In addition, since flow through the exhauster nozzle will normally be choked, the flow rate can be estimated.

Locations of pressure taps and temperature probes are indicated in Figure 2 for the flow measurement nozzle, in Figure 3 for the removable standard orifice installation, and in Figure 4 for the labyrinth leak rate measurement orifice. Both standard orifices are fitted with vena contracta and flange taps. Additionally, for both standard orifices, the lengths of straight pipe upstream and downstream of the orifices are indicated. Table I lists applicable standards for obstruction free, straight lengths of pipe upstream and downstream of the orifices and for pressure tap locations, together with the actual dimensions of the two orifices used with the Transonic Turbine Test Rig. Both installations are well within prescribed values for standard orifice installations.

Pressures were measured in inches of mercury with atmospheric pressure as reference; differential pressures were measured in inches of water. Manometers used had 0.1 in graduations, with an accuracy of $\pm$ 0.03 inches. The accuracy of temperature measurements was taken as $\pm$ 1.0 deg Rankine.

4. PROCEDURE:

Flow nozzle calibration runs were made at essentially constant supply pressures of either 30 psia or 45 psia. Flow rate was controlled by using the valve at the exit of the removable standard orifice installation to set

the differential pressure across the vena contracta taps; supply pressure was maintained by adjusting the Allis Chalmers compressor installation main discharge (butterfly) valve. Differential pressures were increased in increments of ten inches of water for increasing flow rates to the maximum differential pressure, then decreased an increment of five inches of water, followed by incremental reductions of ten inches of water to minimum flow. Approximate ranges of flow rates of the tests were 1.44 to 3.67 lbm/sec at 30 psia and 1.8 to 4.6 lbm/sec at 45 psia.

Plenum labyrinth leak tests were made by varying the supply pressure from about 24 to about 44 psia. Pressures were set by observing the turbine plenum pressure reading. The pressure was varied by about five inches of mercury increments on pressure increase, decreasing from maximum by about 2.5 inches of mercury and then incremental decreases of about five inches of mercury. Tests were made without the exhauster and associated piping installed. Hence the maximum ratio of plenum supply pressure and labyrinth discharge pressure was limited to approximately 3.

5. FORMULAS:

Square-Edged Orifice and Flow Nozzle

The basic flow equation for a square-edged orifice or flow nozzle, in lbm/hr, is, from page 6, Reference 2:

$$W = 359.1\, D_2^2\, \alpha K Y_1 \sqrt{\rho_1 h_w} \tag{1}$$

where:

D_2 = orifice diameter or nozzle throat diameter (inches)

α = coefficient of thermal expansion based on D_2^2 (dimensionless)

[illegible]

K = discharge coefficient (dimensionless)

Y_1 = expansion coefficient (dimensionless)

ρ_1 = fluid density (lbm/ft^3) at upstream tap

h_w = differential pressure across taps (in H_2O)

The equation of state for an ideal gas is:

$$\rho_1 = \frac{p_1 \, (144)}{RT_1} \tag{2}$$

where p_1 is in psia, $R = 53.35$ ft lb/lbm^{o}R, and T_1 is in ^{o}R. For P_1 in in. Hg, equation (2) becomes:

$$\rho_1 = \frac{P_1 \, (0.4912) \, (144)}{(53.35) \, T_1} = 1.328 \, \frac{P_1}{T_1} \tag{2a}$$

Inserting (2a) in (1), and converting the latter to lbm/sec:

$$w = \frac{359.1}{3600} \, D_2^2 \, \alpha K Y_1 \, \sqrt{1.328 \, \frac{P_1 h_w}{T_1}} \tag{3}$$

or:

$$w = 0.115 \, D_2^2 \, \alpha K Y_1 \sqrt{\frac{P_1 h_w}{T_1}} \tag{3a}$$

The coefficient, α, is determined from the linear coefficient of thermal expansion (α_ℓ). For D_2' = orifice diameter at elevated temperature:

$$\left(\frac{D_2'}{D_2}\right)^2 = \frac{\left[1 + \alpha_\ell(\Delta t)\right]^2}{1} = 1 + 2 \, \alpha_\ell \, (\Delta t) + \alpha_\ell^{\,2} \, (\Delta t)^2 \tag{4}$$

Neglecting the higher order final term and letting $(D_2')^2 = \alpha(D_2)^2$:

$$\alpha(D_2)^2 = \left[1 + 2 \, \alpha_\ell \, (\Delta t)\right] \, D_2^2 \tag{5}$$

and

$$\alpha = 1 + 2 \, \alpha_\ell \, (\Delta t) \tag{6}$$

For both standard orifices, made of type 304 stainless steel, $\alpha_\ell = (9.5) \, 10^{-6} \, {}^{o}F^{-1}$, which gives:

$$\alpha_o = 1 + 0.0019 \left(\frac{t-68}{100}\right) \tag{7}$$

(The value 0.00193 vice 0.0019 was used in calculations, the former value being determined from Figure 98, p. 257, Ref. 2). For the flow nozzle made of 2024-T4 aluminum, α_ℓ = (12.6) 10^{-6} $^oF^{-1}$, and

$$\alpha_n = 1 + 0.00252 \left(\frac{t-68}{100}\right) \tag{8}$$

The discharge coefficient, K, is dependent on the ratio of orifice or nozzle diameter to upstream pipe diameter (i.e., $\beta = D_2/D_1$) Reynolds Number based on D_2, and the type of measuring taps used. To simplify computations for standard orifices, the following relation may be used (see p. 65, Ref. 2)

$$\frac{K}{[1+A/R_e]} = \frac{K^*}{[1+A/R_e^*]} \tag{9}$$

where, from page 212, Ref. 2:

$$A = D_2\ (830-5000\beta+9000\beta^2-4200\beta^3+B) \tag{10}$$

with:

$$B = 530/\sqrt{D_1} \qquad \text{for flange taps} \tag{11}$$

$$B = 530/\sqrt{D_1}-100 \qquad \text{for vena contracta taps} \tag{11a}$$

and, from p. 64, Ref. 2:

$$R_e = \frac{6.316W}{D_2 Z'} \tag{12}$$

where Z' is absolute viscosity in centipoises at flow temperature and pressure. For Z' independent of pressure and assuming a linear variation of Z' in the temperature range from 50°F to 300°F, there is from p. 335, Ref. 2:

$$Z = 100Z' = 1.9 + 0.24 \left(\frac{t}{100} - 1\right) \tag{13}$$

Equation (13) in equation (12), with W=3600w, and defining X=Re x10^{-6}, yields:

$$X = \frac{(6.316)\ (3600)\ w\ (100)\ \ (10^{-6})}{D_2 Z} = \frac{2.275w}{D_2 Z} \qquad (14)$$

Letting, $\zeta = 1 + A/_{Re}$;

$$\zeta = 1 + \frac{A\ 10^{-6}}{X} \qquad (15)$$

Equation (9) becomes:

$$\frac{K}{\zeta} = \frac{K*}{\zeta^*} \qquad (16)$$

For flange taps, values of $K* = K_\infty$, corresponding to $Re^* = \infty$, are tabulated in Table 19 of Ref. 2. Equation (16) is then:

$$K = \zeta K_\infty \qquad (16a)$$

For vena contracta taps, this shortcut method is not authorized and values of K_∞ are not tabulated. With K tabulated as a function of Re in Table 20 of Ref. 2, an iterative procedure using these variables is normally necessary, with flow rate, w, as a third variable. However, by judicious choice of Re^* and, hence, K^*, equation 16 becomes for vena contracta taps:

$$K = \frac{\zeta}{\zeta^*} K* \qquad (16b)$$

For the removable standard orifice installation, Re* was chosen as (10^6), giving K*=0.698. For a Reynolds Number of $2(10^5)$, the value of K determined from equation (16b) differs by 0.114 percent from the tabulated value, while the range of Reynolds Numbers for tests was approximately: $3.9\ (10^5)$ to $1.3\ (10^6)$.

Similarly, for the two inch pipe, square-edged orifice, Re* was selected as (10^5), with K* = 0.6103. At a Reynolds Number of $5(10^4)$ the computed value of K differs from the tabulated value by about 0.1 percent and at a Reynolds Number of $2(10^5)$ the difference is about 0.13 percent. The range of Reynolds Numbers in the tests conducted to date was approximately $5(10^4)$ to $1.2(10^5)$.

[illegible]

Tests yet to be conducted with the exhauster installed will be at somewhat higher Reynolds Numbers than $1.2(10^5)$. However, since it is anticipated that the labyrinth seal flow will become choked at pressure ratios (plenum total to seal discharge) only slightly higher than 3:1, it is considered that $Re=2(10^5)$ is a realistic upper limit.

The expansion factor, Y_1, which compensates for compressibility in the flow through the orifice, is defined by equation (60), p. 71, Ref. 2, as:

$$Y_1=1-(0.41 + 0.35\beta^4)\frac{\Delta p}{\gamma p_1} \tag{17}$$

where Δp and p_1 must be in consistent units, and γ is the specific heat ratio. With $\gamma = 1.4$, $\Delta p = h_w$ (in H_2O) and $p_1 = P_1$ (in Hg), equation (17) becomes:

$$Y_1=1.0-\frac{(0.41 + 0.35\beta^4)}{(1.4)\ (13.59)}\ \frac{h_w}{P_1} \tag{17a}$$

Calculations required to put equation (3a) in the form:

$$w = C\alpha\zeta Y_1\sqrt{\frac{P_1h_w}{T_1}} \tag{18}$$

for each standard orifice were accomplished by using the CDC 1604 computer. Additionally the several constants in the expressions for α, Y_1, and X were computed. Table II lists the values to be used for each standard orifice.

It is to be noted that equation (16a) was used for flange taps, and equations (16b), (14) and (15) were used for vena contracta taps, along with appropriate values of K_∞, K^* and Re^*. In solving equation (18), $\zeta=1$ is assumed and w is computed. With this value of w, ζ is obtained from equations (14) and (15), and equation (18) is again solved using the corrected value of ζ.

The object of the nozzle calibration was the determination of the discharge coefficient K. Hence, equation (3a) is rewritten as:

$$K_n= \frac{w\sqrt{T_1}}{0.115\ D_2^2\ \alpha_n Y_1\sqrt{P_1h_w}} \tag{19}$$

where w is the flow rate as measured by the standard orifice, α_n is given by equation (8) and Y_1 by equation (17a).

Shaft Labyrinth Seal Leak Rate

The basic method for estimating the shaft labyrinth seal leak rate is that developed by Egli in Ref. 3. The equation of continuity for an ideal labyrinth seal (i.e., all kinetic energy destroyed after each throttling) is, from Ref. 3:

$$w = \alpha \frac{A\ Po}{\sqrt{To}} \sqrt{g/R}\ \varphi \tag{20}$$

where:

$$\varphi \equiv \left[\frac{1-(P/p_o)^2}{n + (2/\gamma)\ln(po/p)}\right]^{1/2} = \left[\frac{1-r^2}{n+\frac{2}{\gamma}\ \ln(1/r)}\right]^{1/2} \tag{21}$$

α= discharge coefficient for a single throttling (dimensionless)

A= cross-sectional area of seal passage (in^2)

P_o= total pressure at seal entrance (psia)

p = static pressure at seal discharge (psia)

R= gas constant for air (ft lb/lbm^{o}R)

To= total temperature at seal inlet (^{o}R)
(Total temperature is constant across seal)

n= Number of throttlings

r= overall seal pressure ratio (p/p_o)

The dimensionless seal pressure ratio function, φ, was developed by series expansion with a lower limit of pressure ratio across any one throttling of 0.8. Since a preliminary analysis of exhauster performance showed an overall seal pressure ratio of about 0.55 (hood case pressure to ambient pressure), this same assumption was made for the ten throttlings through the shaft labyrinth seal.

To account for a non-ideal labyrinth, a carryover factor, γ^*, is introduced to adjust for carryover of kinetic energy from one throttling to the next. Equation (20) then becomes:

[illegible]

[illegible]

[illegible]

[illegible]

$$w=\alpha\gamma^* \frac{Ap_o}{\sqrt{T_o}} \sqrt{g/_R}\ \varphi \tag{22}$$

where

$$\alpha=f(\delta/t, t) = \text{constant}$$

$$\gamma^* = f(\delta/s, n) = \text{constant}$$

Here, δ = tooth clearance, t=tooth width and s=chamber width (see Fig. 5).

Unfortunately, the curves of α versus δ/t given in Ref. 3 make determination of α for the geometry of the shaft seal somewhat uncertain. Hence, the value of α=0.76 was taken from experimental data given on page 72, Ref. 4, where α is plotted as a function of δ/t and δ/b, where b is the chamber heighth.

Plots of γ^* vs δ/s given in Ref. 3 are based on experimental data for $\delta > 0.01$. As cited in that reference, Friedrich's tests of straight through labyrinths with δ= 0.006 to 0.010 inches, and with two and three throttlings, indicated somewhat smaller values of γ^* than those given in Ref. 3 for the geometry of the shaft seal of the Transonic Turbine Test Rig. Considering, also, ten rather than two or three throttlings and data for multiple throttlings given in Ref. 4, Friedrich's value of γ^* = 1.15 was selected in preference to Egli's value of γ^*= 1.25.

With these values of α and γ^*, the final formula for estimating shaft seal leakage is, with: $A= \pi\ 1.2 \times .005$:

$$w= 0.0128 \frac{p_o}{\sqrt{T_o}} \varphi \tag{23}$$

With w^* defined by

$$w^* = \frac{w\sqrt{T_o}}{A\,P_o} \sqrt{R/g} = 0.874\ \varphi \tag{24}$$

a single plot of w^* versus overall seal pressure ratio (r) can be used since:

$$\varphi = f(r)$$

<u>Plenum Labyrinth Seal Leak Rate</u>

Since plenum labyrinth leak rates were determined experimentally, use of equation (22) was limited to attempts to correlate experimental data with the

[illegible]

method and to extrapolate higher overall seal pressure ratio data (0.34 to 0.6) to lower pressure ratios (0.16 to 0.34). For the two identical labyrinths on either side of the plenum, equation (22) is written:

$$w^* = \frac{w\sqrt{To}}{A\ P_o}\ \sqrt{R/_g} = 2K\varphi \qquad (25)$$

where K=$\alpha\gamma^*$ of equation (22). Then, if α and γ^* (or $\alpha\gamma^*$) are in fact constant, K=constant, and experimental data at higher values of r can be extrapolated to lower values of r. Since it is anticipated that flow through the seal will be choked at φ_{max} = 0.2798 (r= 0.225), the range of extrapolation would be relatively small.

6. RESULTS:

Flow Nozzle Calibration

Since the tolerance for vena contracta taps is less than that for flange tops (see Section 8), plotted and tabulated data are vena contracta values unless otherwise indicated.

Figure 6 shows plots of nozzle discharge coefficient, Kn, versus Reynolds Number determined from tests of 8 September and 13 September 1965. Figure 6 indicates that Kn can be taken as a constant above Reynold's Numbers of about $6(10^5)$ - (w≅2lbm/sec). By the method of least squares, values of Kn=constant were determined for the following data groups: 8 September tests at 30 psia; 13 September tests at 30 psia; these two tests combined; 13 September tests at 45 psia; and all the above-listed tests of 8 and 13 September combined. Resulting minimum error values of Kn=constant are tabulated in Table III with the least squares error. In addition, certain values of Kn=constant are shown in Figure 6. Values in Table III show reasonably close agreement. However, the 13 September test, vena contracta values are considered preferable, since vena contracta flow rates are generally more accurate than flange tap flow rates, and since some difficulty was encountered with water collecting in orifice pressure tap lines during the 8 September tests. (Even though these

lines were continually drained during the 8 September tests, the data are considered inferior to those from the 13 September tests, during which the water collection problem did not exist.)

Shaft Labyrinth Seal Leak Rate

Figure 7 shows w* and φ plotted versus overall seal pressure ratio (r)- see equation (21) - for use in estimating shaft seal leakage.

Plenum Labyrinth Seal Leak Rate

Experimentally determined values of K and w* from equation (25) are plotted versus overall seal pressure ratio in Figures 8 and 9 respectively. It is apparent from Figure 8 that extrapolation of K to lower pressure ratios would be of questionable accuracy. To provide a mathematical expression for w*=f(r), the data shown in Figure 9 was fit with a parabola by the method of least squares. In this curve fitting, a parabola of the form:

$$r = a + bw^* + cw^{*2} \tag{26}$$

was assumed. In addition to experimental data, the known condition that at r=1.0, w*=0.0, was introduced. Symmetry about the r axis was assumed by including in the least squares formulation both plus and minus values of w* for each data point used (i.e., $b \equiv 0.0$).

During the 26 October tests, especial care was taken to assure flow stabilization before data was taken. Although the difference between increasing pressure and decreasing pressure data was not eliminated, it was materially reduced from this difference for the 20 October tests. Consequently, the decreasing pressure data (points to the far right in Figures 8 and 9) were neglected in establishing both the solid curves of Figures 8 and 9 and the least squares parabola.

The resulting least squares second order polynomial, for which the least squares is 0.02079, is:

$$r=1.0-4.0791w^{*2} \tag{27}$$

or, solved for w*:

$$w^* = \sqrt{\frac{1.0-r}{4.0791}} \tag{27a}$$

This equation is plotted in Figures 9 and 10 along with experimental data. Also shown in Figure 10 are segments of three curves, $w^*=2K\varphi$, for values of K appropriate to three different ranges of seal pressure ratio.

7. DISCUSSION AND RECOMMENDATIONS:

Based on the nozzle flow rate experiments conducted, it is recommended that a valve of 0.9992 be used for the nozzle discharge coefficient, Kn, in equation (3a). However, when supply pressures to the nozzle are 30 psia or below a valve of 1.0012 should be used.

Equation (27a) is recommended for determination of plenum labyrinth leak rates in the pressure ratio range: 0.03 to 1.0. Since plenum labyrinth leak rates (0.04 to 0.10 lbm/sec) are very much smaller than turbine flow rates (2.0 to 3.9 lbm/sec), the slight errors in the leakage flow rates caused by using equation (27a) should have a negligible effect on the accuracy of turbine flow rates. Furthermore, this equation should give satisfactory flow rates at higher pressure ratios (see Figure 10).

For exhauster operation with a supply pressure of 45 psia to the exhauster nozzle and the turbine, the estimated shaft seal leak rate given by equation (24) is about 0.003 lbm/sec, while flow rates through the exhauster nozzle and the turbine are estimated as 9.38 and 3.87 lbm/sec respectively. Using Egli's values of nozzle discharge coefficient and carryover factor ($\alpha \simeq 0.9$ and $\gamma^*=1.25$), shaft seal leak rate estimates would be approximately 50% greater than estimates obtained using equation (24). However, plenum labyrinth leak rates predicted using Egli's values of α and γ^* are about 50 to 70 percent greater than the leak rates determined experimentally. Hence, the use of equation (24) is recommended for predicting shaft seal leakage flow rates.

[illegible]

The recommended flow rate equations are summarized in Table IV for ready reference.

8. TOLERANCE IN FLOW MEASUREMENTS

The overall error tolerance in percent in flow measurements using sharp-edged orifices is given by the square root of the sum of the squares of "exponents times tolerances, in percent" ascribable to the individual items in the flow equation. Hence, from page 24, Ref. 2:

$$\Delta(w) = \left\{\left[2\Delta(D_2)\right]^2 + \left[\Delta(\alpha)\right]^2 + \left[\Delta(k)\right]^2 + \left[\Delta(Y_1)\right]^2 + \left[\frac{1}{2}\,\Delta(hn)\right]^2 + \left[\frac{1}{2}\,\Delta(P_1)\right]^2 + \left[\frac{1}{2}\,\Delta(T_1)\right]^2\right\}^{1/2} \qquad (28)$$

Tolerances of individual items for the Transonic Turbine Test Rig installation, based on Reference 2, are given in Table V.

The error in values of discharge coefficients of the flow nozzle and the sharp-edged orifice must be approximately the same. It follows also that the tolerances in flow nozzle flow rate measurements must be the same as those for the sharp-edged orifice used in calibration. Hence, vena contracta tap based values are preferable due to a lower tolerance than that for flange taps.

Tolerances in measured labyrinth leakage rates also should be in accordance with the preceding paragraph. However, it is apparent from Figures 8 and 9 that there is considerable scatter in the data. Hence, a tolerance arbitrarily set at ± 10% would appear reasonable.

Tolerances in estimated shaft seal leak rates are difficult to specify exactly. Here, a reasonable tolerance could be set at ± 20%, based on qualifications set forth in the literature cited.

9. REFERENCES:

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2. Stearnes, R. F., et al., "Flow Measurement with Orifice Meters," Van Nostrand Co., Inc., New York, 1951. (USNPGS Reference Library No. 532.52, S7)

3. Egli, Adolf, "The Leakage of Steam Through Labyrinth Seals," Transactions of the ASME, Vol. 57, 1935, pp 115-122.

4. Jerie, Jan, "Flow Through Straight-Through Labyrinth Seals," Proceedings of the Seventh International Congress of Applied Mechanics, 1948, Vol 2, Part I, (USNPGS Reference Library No. 620.1063, I6, 7th, 1948).

TABLE I

Required and Actual Dimensions

Transonic Turbine Test Rig Square-Edged Orifices

Orifice Installation	Pipe Diam D_1(in)	Orifice Diam D_2(in)	$\beta = D_2/D_1$	Vena Contracta Tap Location -(b)						Flange Tap Location (b)			
				Upstream		Downstream				Upstream		Downstream	
				Req	Act	Min	Max	Rec	Act	Req	Act	Req	Act
Removable	6.065	4.2425	.6995	1	1	.325	.56	.45	.464	1 in	1 in	1 in	1 in
2" pipe	2.067	.825	.3991	1	1	.475	.99	.745	.755	1 in	1 in	1 in	1 in

	Upstream to straightener				Downstream	
	Entrance		Exit			
	Min	Act	Min	Act	Min	Act
Removable	11.5	15.9	6.5	14.42	6 (c)	11.7 (c)
2" Pipe	7.8	16.2	5.2	14.25	3.2	5.8

Notes:

(a) Unless otherwise specified, dimensions are in multiples of 1.

(b) Upstream (Downstream) of the upstream (Downstream) face of the orifice.

(c) To exit valve upstream flange.

TABLE II

Flow Equation Constants - Square-Edged Orifices

Transonic Turbine Test Rig

Orifice Installation	β	Flange Taps C	Flange Taps A	Vena Cont Taps C	Vena Cont Taps A	Cx	Cy
		EQ(18)	EQ(10)	EQ(18)	EQ(10)	EQ(14)	EQ(17a)
Removable	0.6995	1.4257	0.00218 x 10^6	1.4405	0.001756	.5359	0.026
2" pipe	0.3991	0.0476	0.000305	0.04761	0.0002249	2.7561	0.022

EQUATION

(18) $$w = C\alpha\xi Y_1 \sqrt{\frac{P_1 h_w}{T_1}}$$

(10) $$A = D_2(830 - 5000\beta + 9000\beta^2 - 4200\beta^3 + B)$$

(14) $$X = \frac{(6.316)(3600)10^{-4} w}{D_2 Z} = C_x \frac{w}{Z}$$

(17a) $$Y_1 = 1.0 - \left[\frac{(.41 + .35\beta^4)}{1.4 \times 13.59}\right] \frac{h_w}{P_1} = 1 - C_y \frac{hw}{P_1}$$

(13) $$Z = 1.9 + 0.24\left(\frac{t}{100} - 1\right)$$

(8) $$\alpha = 1.0 + 0.00252\left(\frac{t-68}{100}\right)$$

(15) $$\zeta = 1.0 + \frac{A(10^{-6})}{X}$$

TABLE III

Flow Nozzle Discharge Coefficients

Transonic Turbine Test Rig

Test Date	Nominal Pressure	Data Points	Flange Taps		Vena Cont Taps	
			Error #	Kn	Error #	Kn
8 Sep	30 psia	9	.00828	.9994	.00599	1.0038
13 Sep	30 psia	12	.01055	1.0026	.00684	1.0012*
8/13 Sep	30 psia	21	.01531	1.0012	.01071	1.0024
8/13 Sep	30 psia 45 psia	21 12	.01614	1.0011	.01633	1.0012
13 Sep	45 psia	12	.00497	1.0008	.00864	.9992*

Error figures show least squares average difference of data points from Kn for minimum error.

* Recommended values

TABLE IV

Summary of Formulas for Determining Flow Rates

Transonic Turbine Flow Rates

Desired Flow Rate	Basic Formula	Auxiliary Formulas
Nozzle (lbm/sec)	$w=2.073\alpha_n Y_1 \sqrt{\dfrac{P_1\ h_w}{T_1}}$	$\alpha=1.0+0.00252\left[\dfrac{t_1-68}{100}\right]$ $Y_1=1.0-0.023\ \dfrac{h_w}{P_1}$
Plenum Labyrinth Seal Leak Rate (lbm/sec)	$w=0.116\ \dfrac{w^{*}po}{\sqrt{To}}$	$w^{*} = \sqrt{\dfrac{1.0-r}{4.0791}}$ $r = \dfrac{p}{Po}$
Shaft Labyrinth Seal Leak Rate (lbm/sec)	$w = 0.01466\ \dfrac{w^{*}po}{\sqrt{To}}$	$w^{*} = 0.874\ \varphi$ $\varphi = \left[\dfrac{1.0-r^2}{10.+\frac{2}{\gamma}\ln(1/r)}\right]^{1/2}$ $r = \dfrac{p}{po}$; $\gamma = \dfrac{cp}{c_v}$

TABLE V

Percent Tolerances of Flow Equation Variables

and Parameters - Square-Edged Orifices *

Transonic Turbine Test Rig

a) Independent of Orifice:

$\Delta\ (\alpha) = \pm .05$

$\Delta\ (Y_1) = 0 \text{ to } \pm .05$ for $0.0 < \frac{hw}{P_1} < .015$

$= \pm 0.5 \text{ to } \pm 1.0$ for $0.015 < \frac{hw}{P_1}$

$$\Delta\ (hw) = \frac{\pm .03}{hw}\ (100)$$

$$\Delta\ (P_1) = \frac{\pm .03}{P_1}\ (100)$$

$$\Delta\ (T_1) = \frac{\pm 1.0}{T_1}\ (100)$$

b) Removable Standard Orifice Installation:

$$\Delta\ (D_2) = \pm \frac{.0005}{4.2425}\ (100) = \pm .0118$$

$\Delta\ (C) = \pm 1.0$ flange taps

$= \pm 0.4 \text{ to } \pm 0.6$ vena contracta taps

c) 2" Pipe Orifice

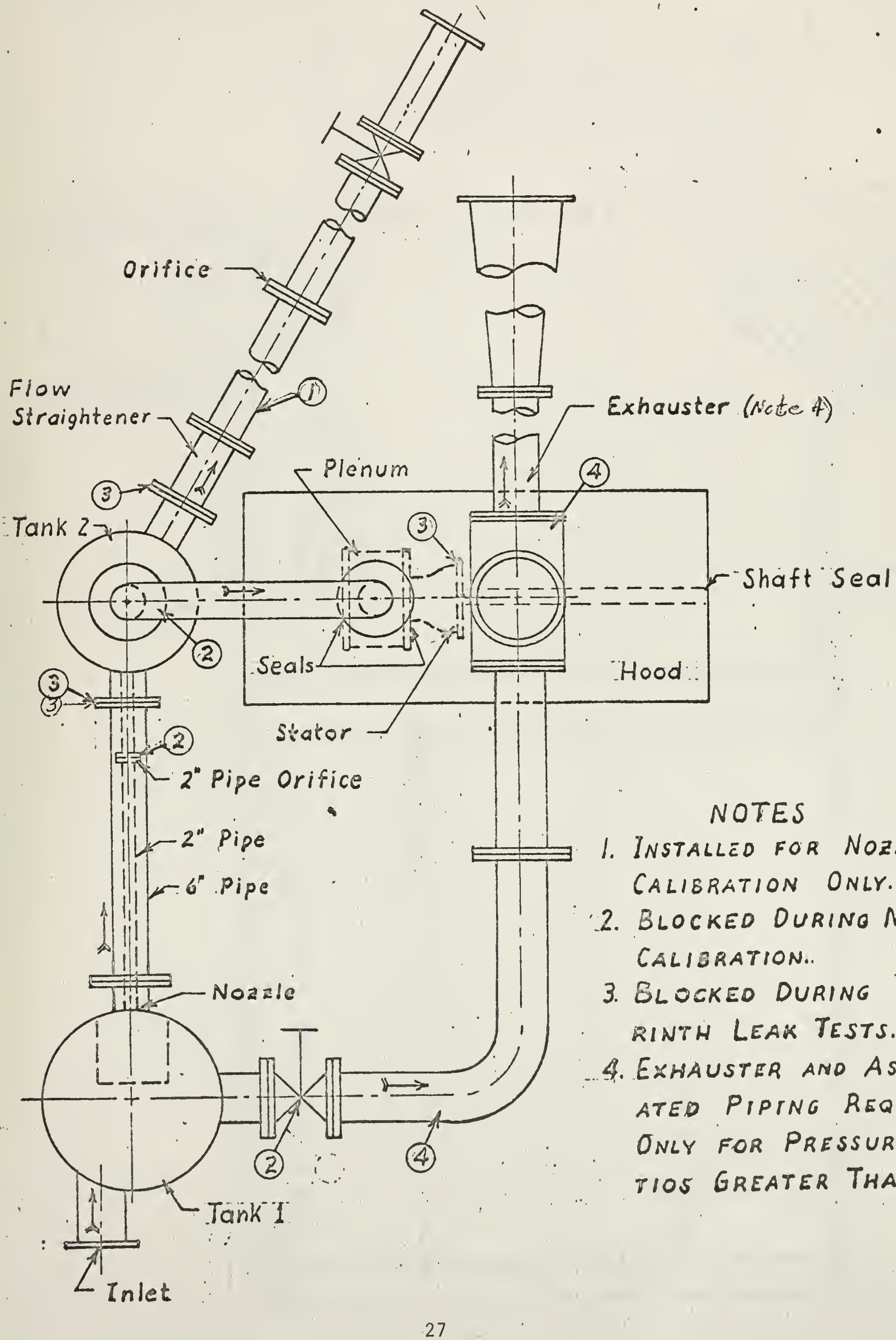
$$\Delta\ (D_2) = \pm \frac{.0005}{.825}\ (100) = .0606$$

$\Delta\ (C) = \pm 1.0$ flange taps

$= \pm 0.4 \text{ to } \pm 0.6$ vena contracta taps

* Basis: Ref. 2

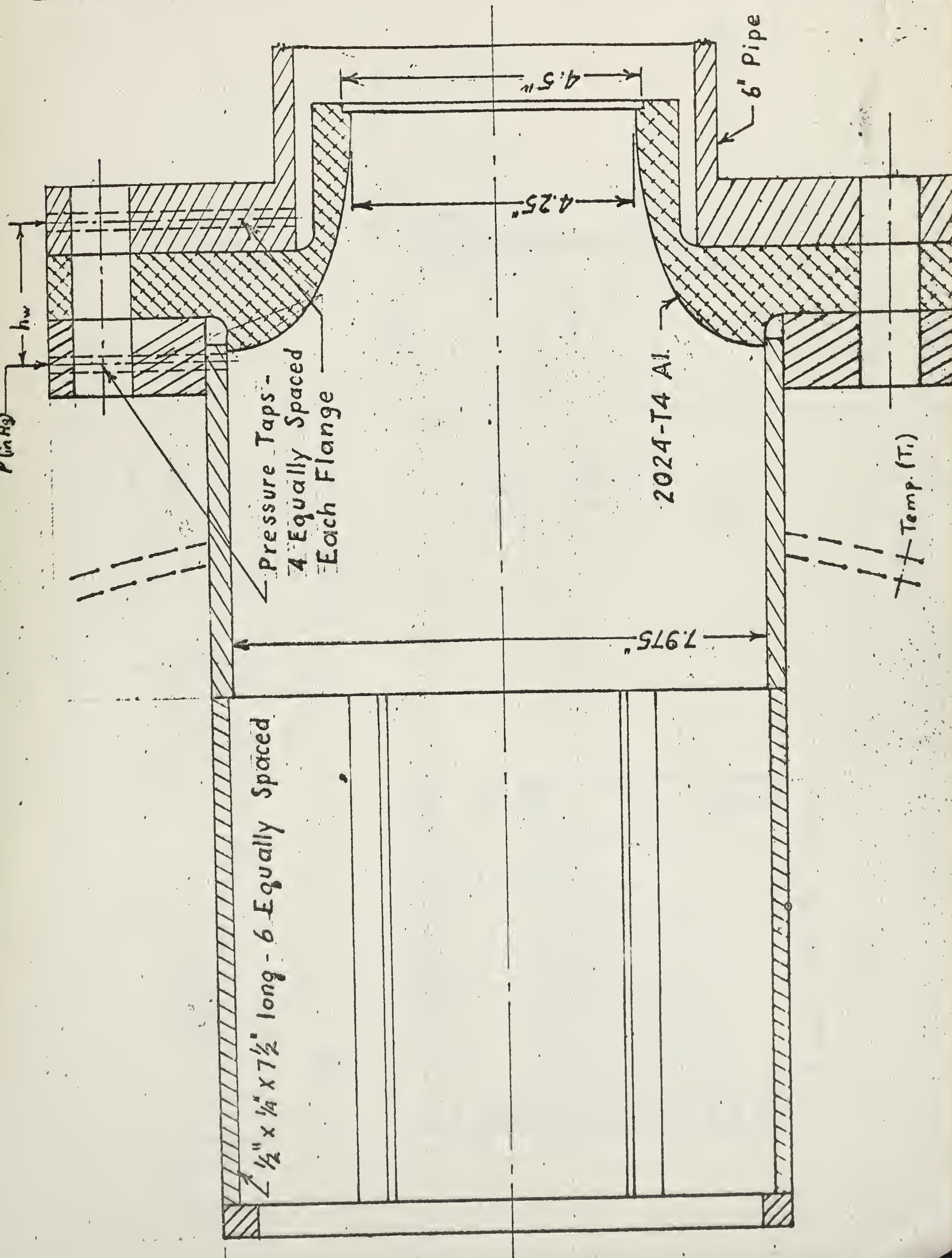
ECKERT	U. S. Naval Postgraduate School	FIG. 1
	Piping Installation	TECH. NOTE
JAN 66	Transonic Turbine Test Rig	NO. 66T-1



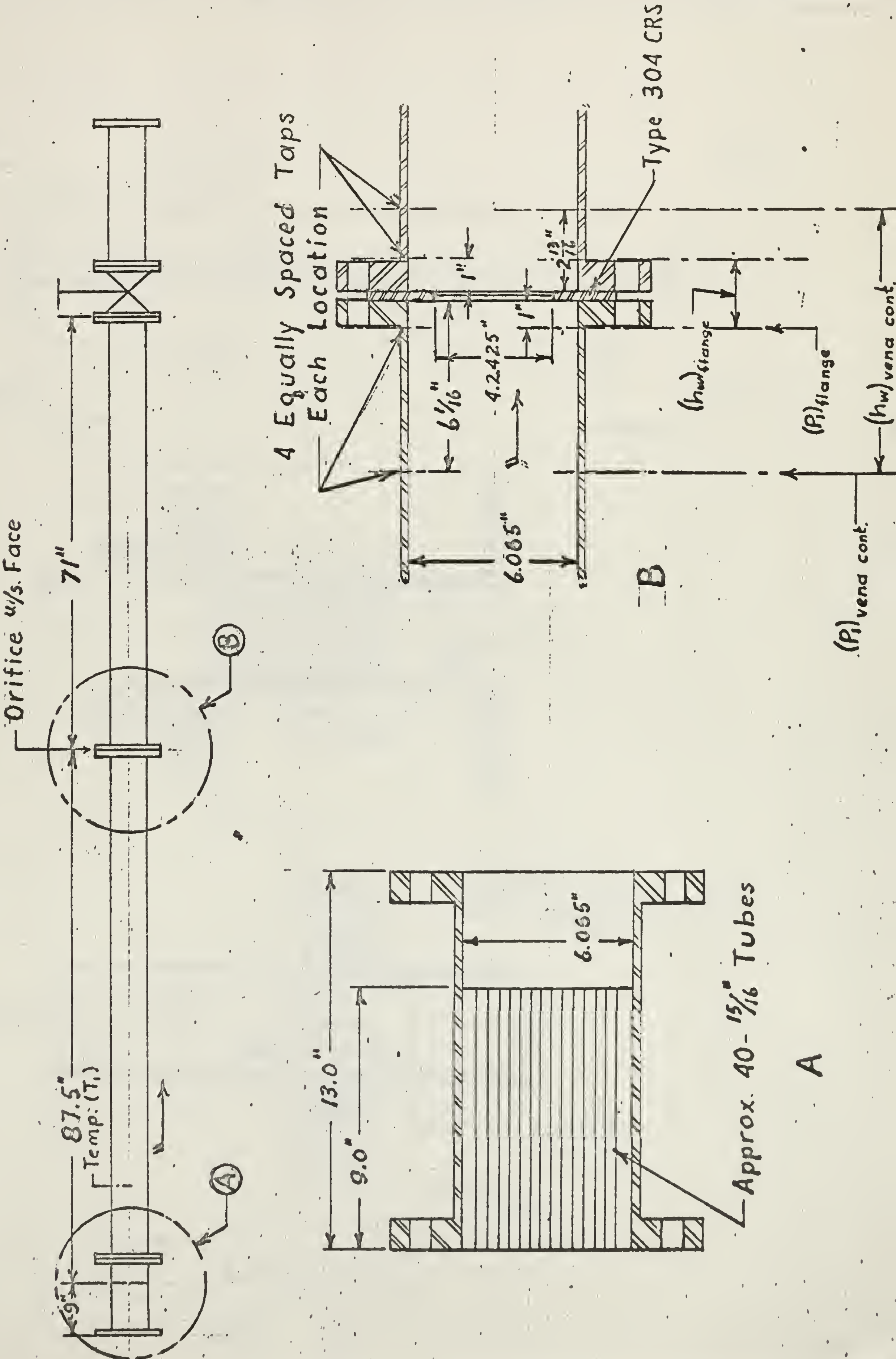
NOTES

1. INSTALLED FOR NOZZLE CALIBRATION ONLY.
2. BLOCKED DURING NOZZLE CALIBRATION.
3. BLOCKED DURING LABYRINTH LEAK TESTS.
4. EXHAUSTER AND ASSOCIATED PIPING REQUIRED ONLY FOR PRESSURE RATIOS GREATER THAN 3:1.

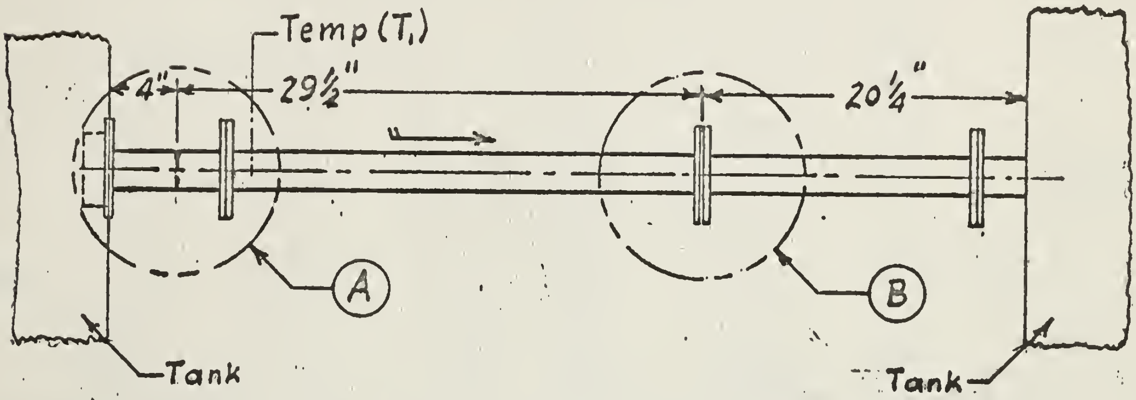
ECKERT	USNAVPGSCOL - Dept of Aeronautics - Propulsion Lab.	FIG. 2
	Flow Measurement Nozzle	TECH NOTE
JAN 66	Transonic Test Rig	NO. 66T-1

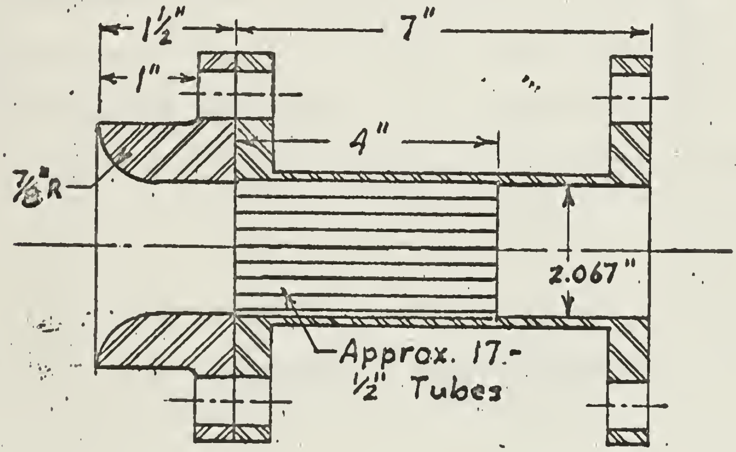


ECKERT	USNAVPGSCOL - Department of Aero.- Propulsion Lab.	FIG. 3
	Removable Standard Orifice Installation	TECH NOTE
JAN 66	Transonic Turbine Test Rig	NO. 66T-1

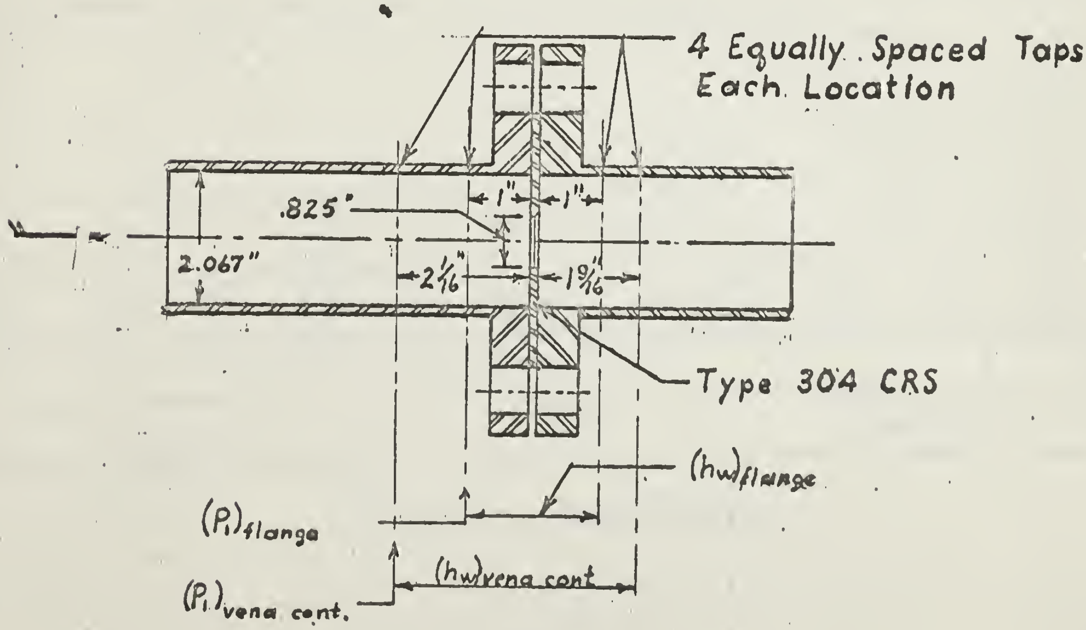


ECKERT	USNPGS DEPT OF AERONAUTICS, PROPULSION LAB	FIG. 4
	2" PIPE STANDARD ORIFICE INSTALLATION	TECH NOTE
JAN 66	Transonic Turbine Test Rig	NO. 66T-1



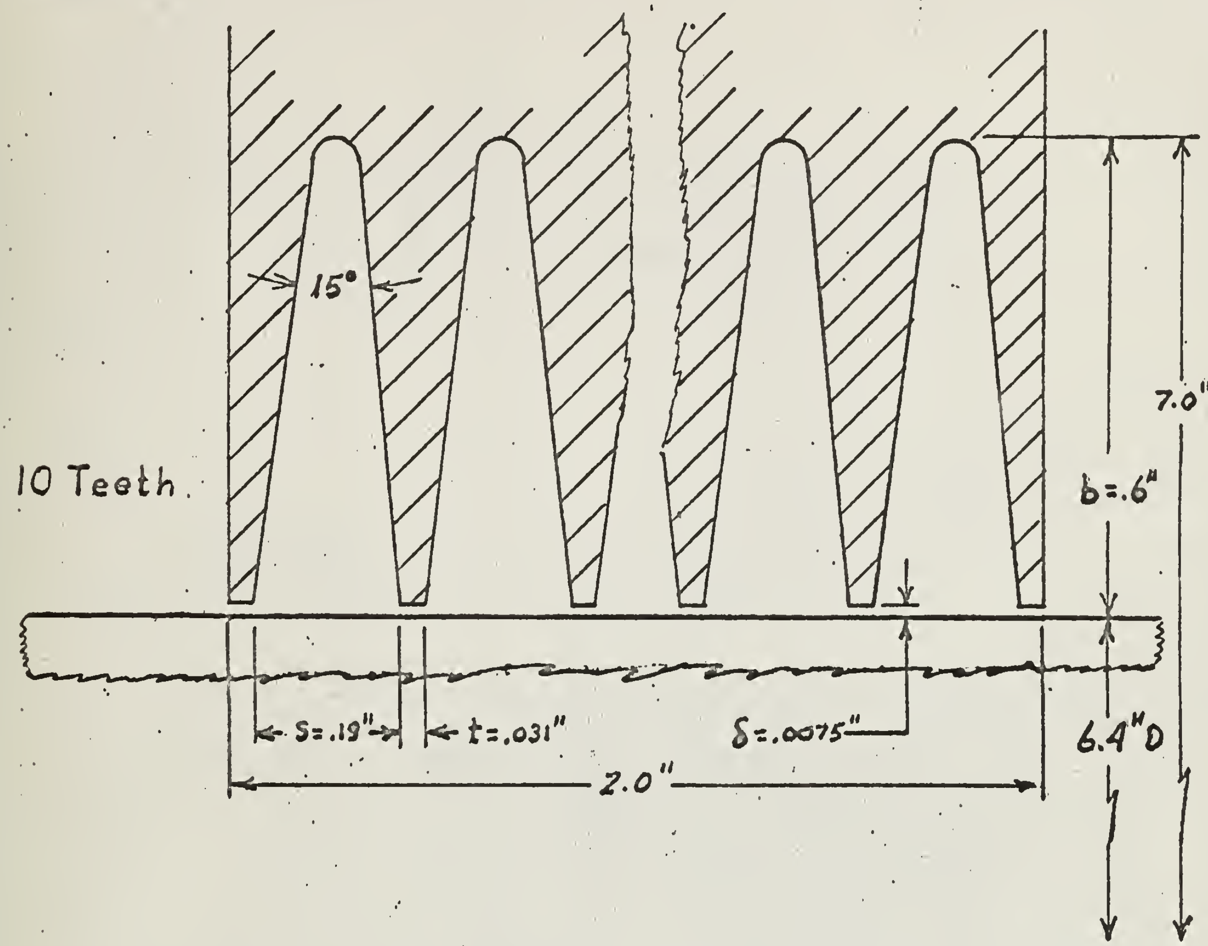


DETAIL A

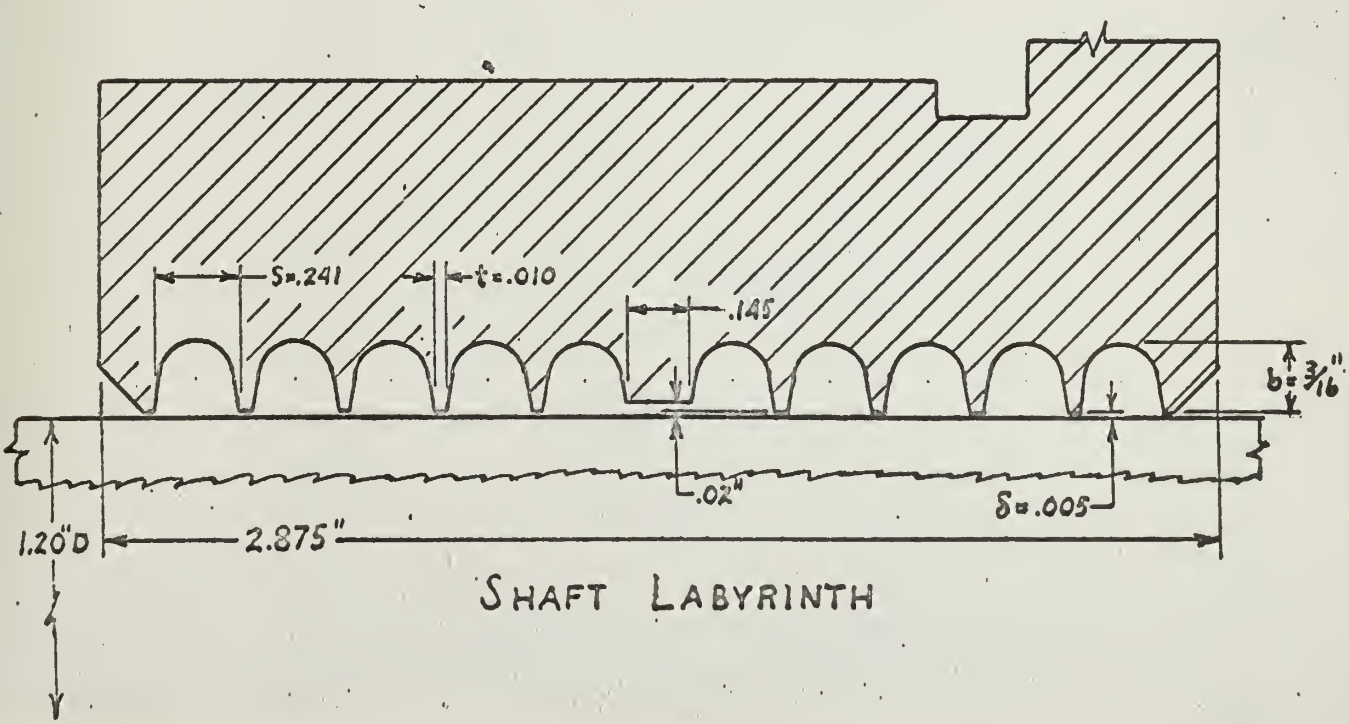


DETAIL B

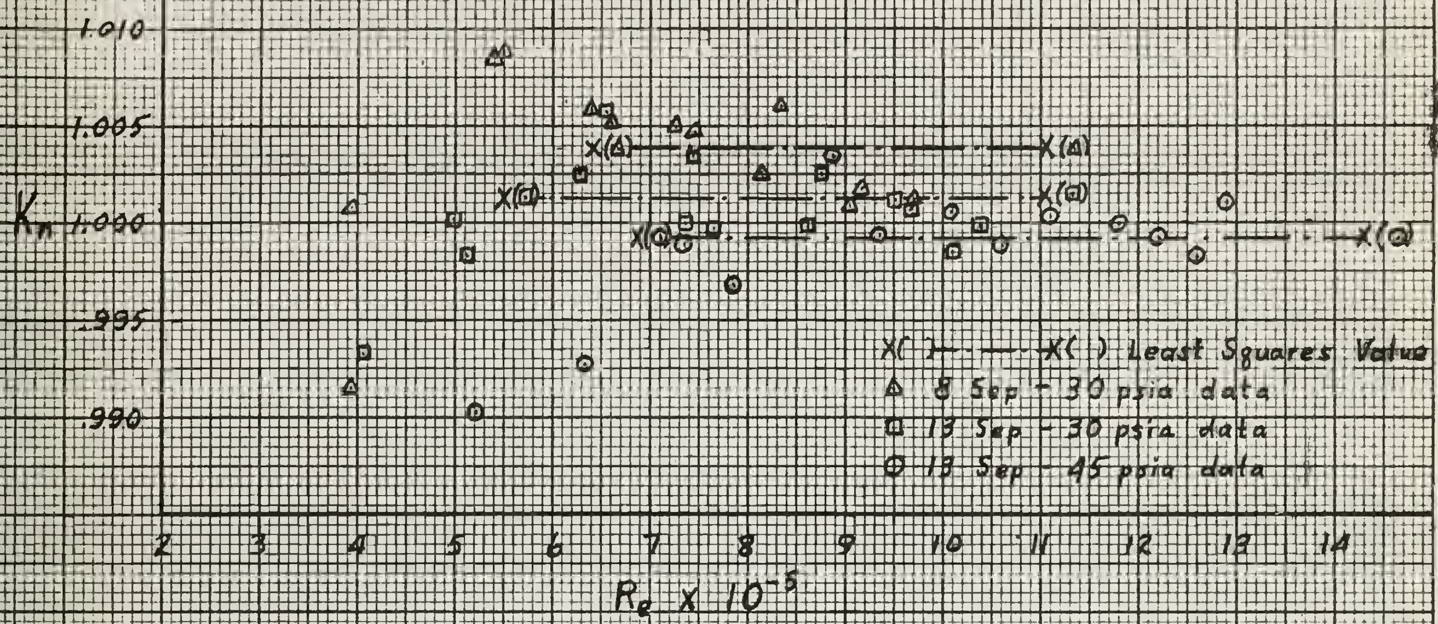
ECKERT	USNPGS DEPT OF AERONAUTICS, PROPULSION LAB	FIG. 5
	Labyrinth Seals	TECH NOTE
JAN 66	Transonic Turbine Test Rig	NO. 66T-1



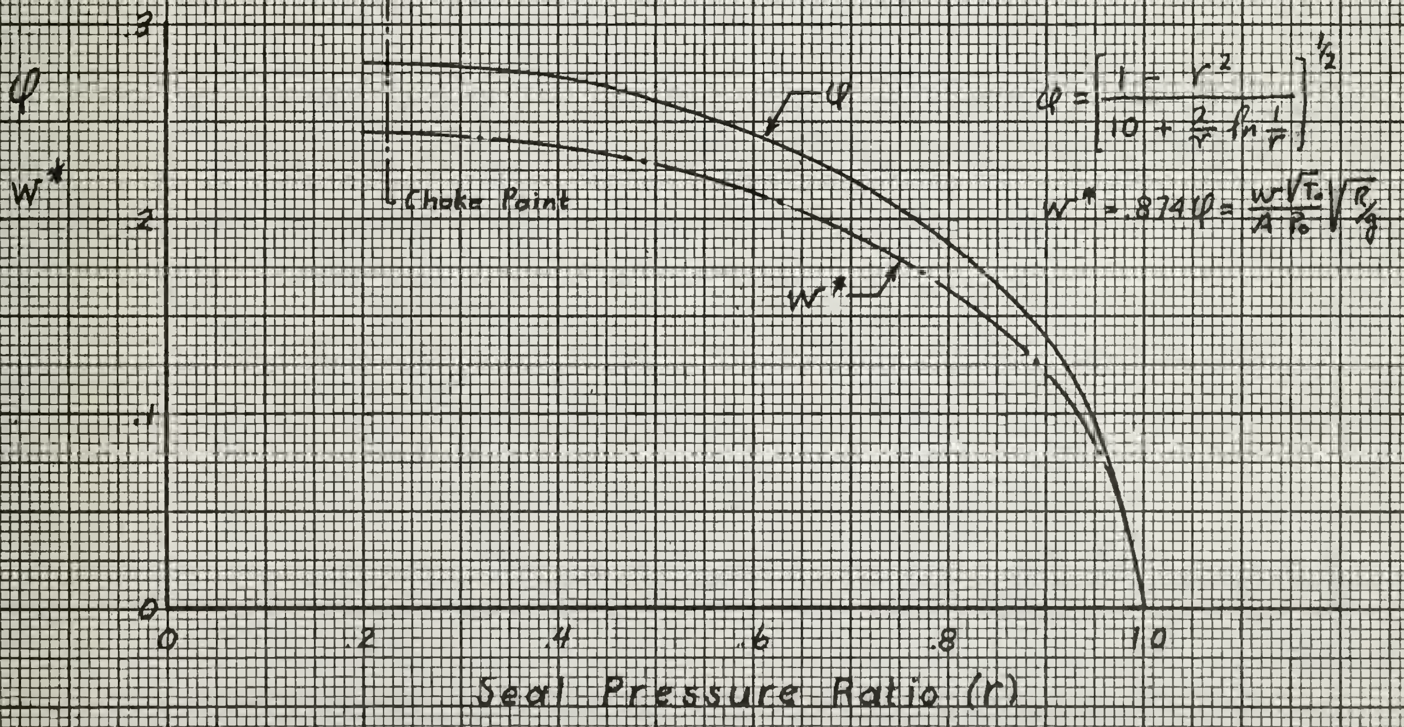
PLENUM LABYRINTH



SHAFT LABYRINTH



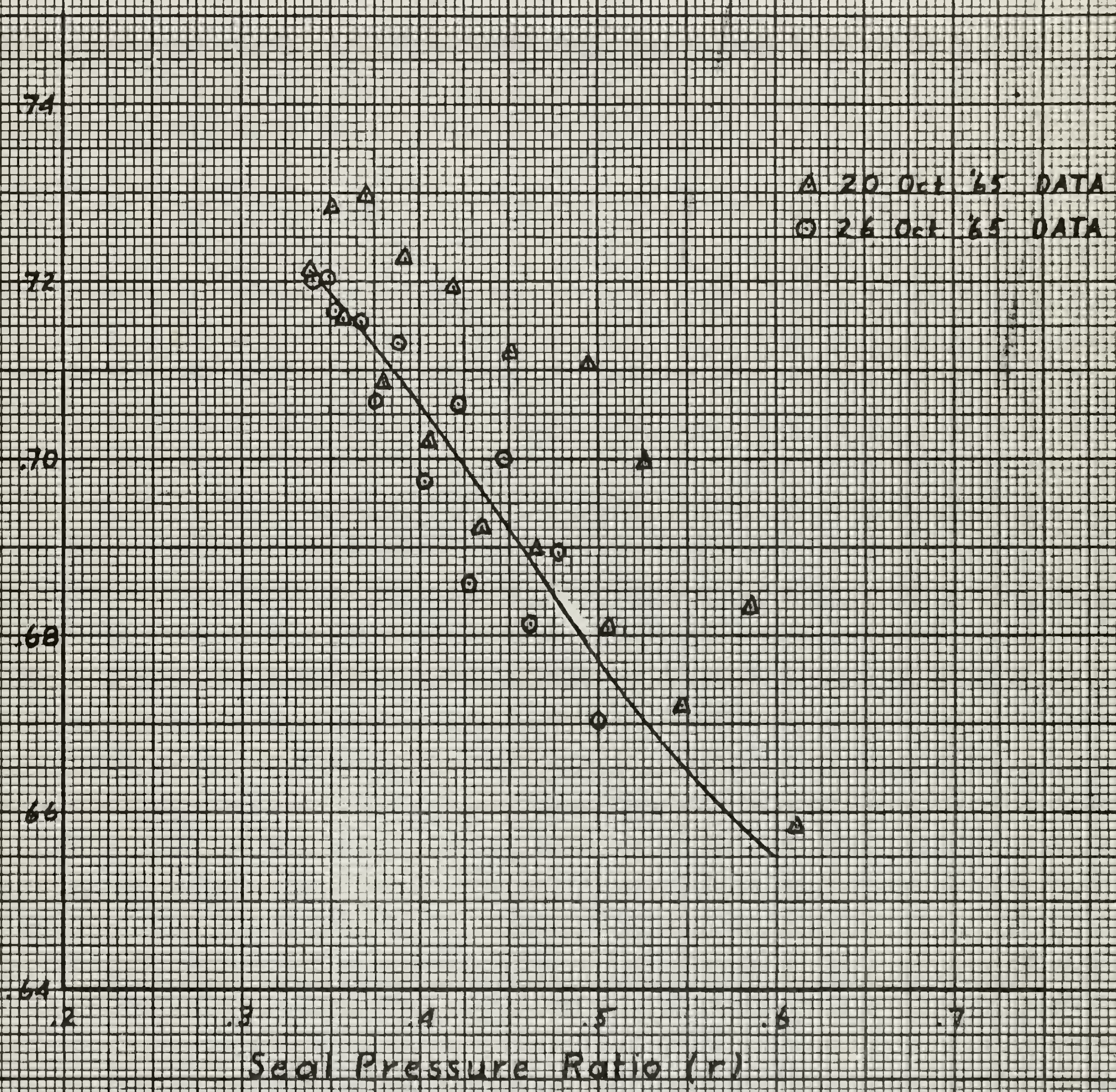
FLOW NOZZLE DISCHARGE COEFFICIENT
TRANSONIC TURBINE TEST RIG
FIGURE 6



SHAFT SEAL FLOW RATE
TRANSONIC TURBINE TEST RIG

FIGURE 7

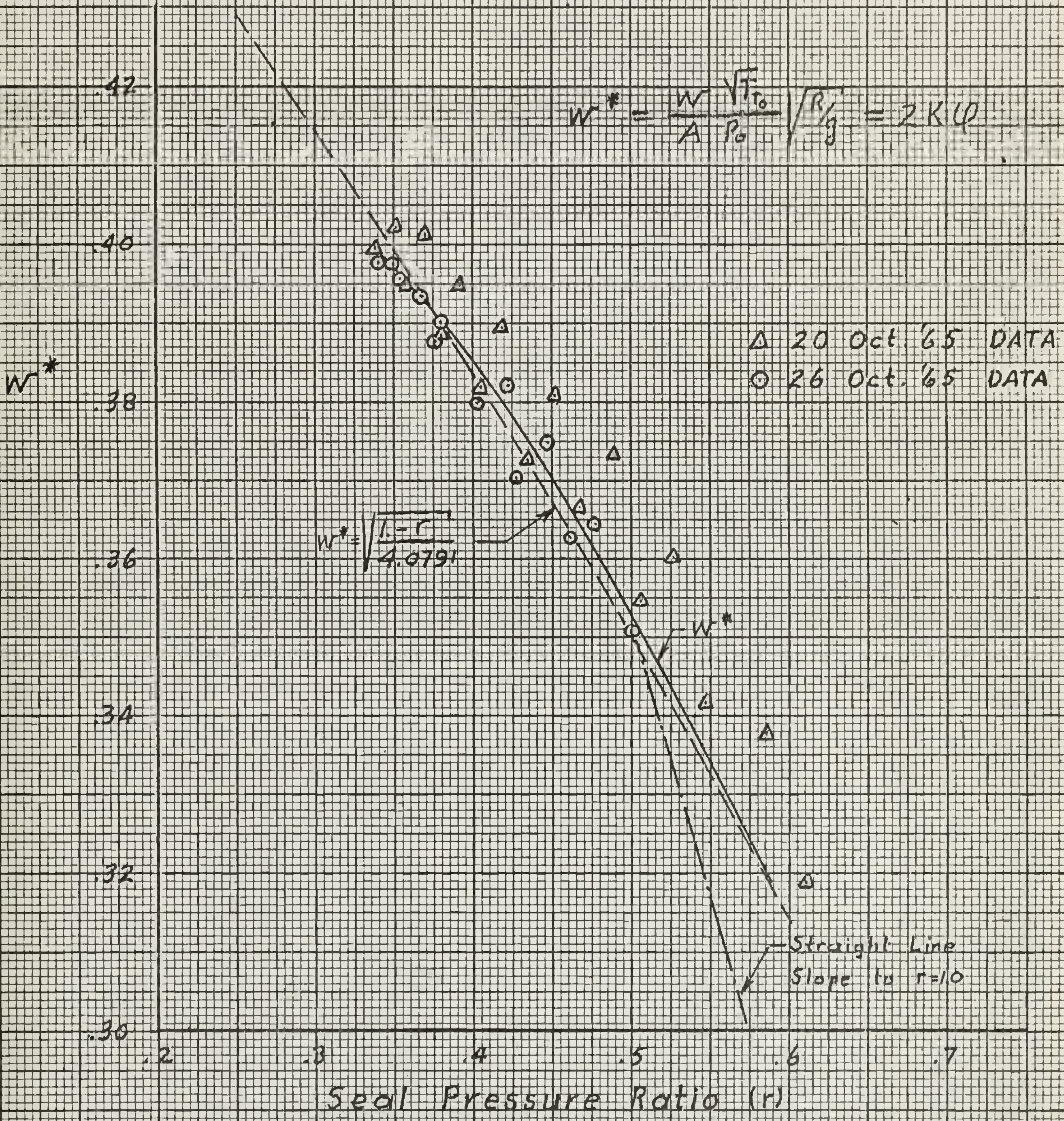
RHE 1/66



PLENUM LABYRINTH SEAL DISCHARGE COEFFICIENT
TRANSONIC TURBINE TEST RIG

Figure 8

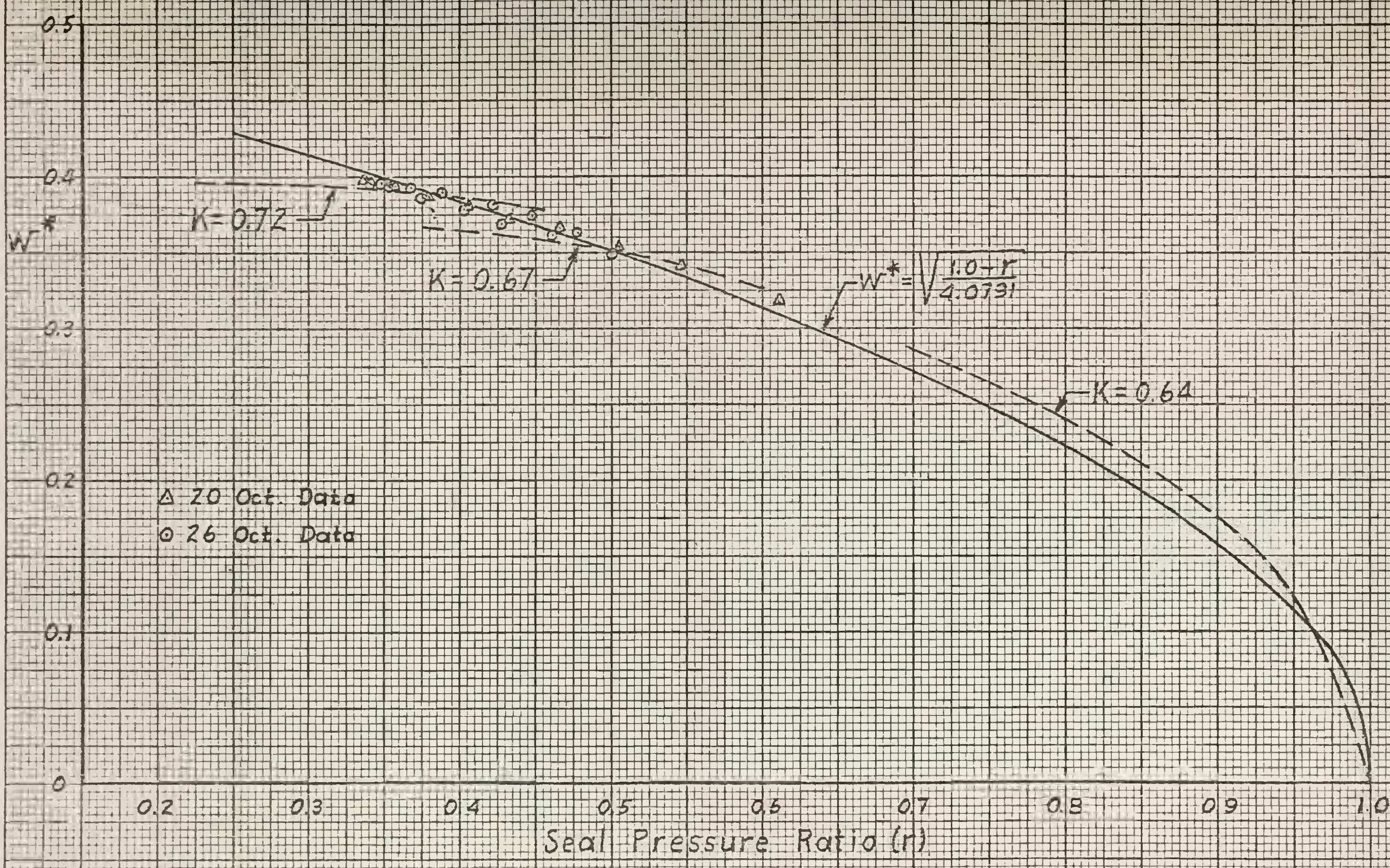
RAE 1/66



PLENUM LABYRINTH SEAL LEAK RATE
TRANSONIC TURBINE TEST RIG

Figure 9

RHE 1/66



LEAST SQUARES PARABOLA FOR PLENUM LABYRINTH LEAK RATE

TRANSONIC TURBINE TEST RIG

FIGURE 10

RHE 1/66

ACCESSION NO.

U-96022

Flow rate determination transonic turbine test rig nozzle calibration